The dependence of the edge and the bet sum on the bet odds.

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Here we want to build a plausible model of the dependence of the edge value on the bet odds, in order then to build the dependence of the bet amount on the odds. For a bet with decimal odds 2, the maximum possible advantage (edge) is 100%, and for a bet with decimal odds equal 1.1, the maximum possible advantage (edge) is only ~10%. That is, the range of possible values of the edge for different betting odds (coefficients) is different, and accordingly, the real edge is likely to be different too. How to reasonably define this dependency? One of the options is offered below.

Let's assume that a player gets the same average profit on bets with different odds. This is similar to the 'fixed profit' strategy. But unlike this strategy, we fix not the potential profit on the bet, that is, the profit that we will get if we win the bet, but the average expected profit on the bet. Consider situation in more details:

MO (mathematical expectation) of net profit on a bet is W = r*S, where
r is the bettor edge,
S is the amount of the bet (turnover per bet).

But the optimal bet sum (according to Kelly) is $rB/(k-1)$,
where k is the decimal odds - bet coefficient,
B is the value of the game bank.

Therefore, the net expected (MO) net profit on one bet will be equal to $r^2B/(k-1)$ for an optimal game. It can be assumed that this value characterizes the effectiveness of the bettor's game. If we assume that it is constant for the player, more precisely, that he is able to achieve and achieves the same efficiency of the game with any coefficient, then this formula determines the dependence of the advantage on the bet coefficient and, at the same time, the dependence of the bet amount on the odds (coefficient). At least, this hypothesis has the right to exist (and be tested). It is qualitatively consistent with the obvious fact that the maximum possible advantage decreases with a decrease in the bet odds. And, for example, the hypothesis of the constancy of the edge regardless of the coefficient does not agree with this fact.

Let R be the player's advantage when coefficient (decimal odds) is of 2. Then on other coefficients it will be equal to $R\sqrt{(k-1)}$. Below is a table of the dependence of the edge on the bet coefficient, provided that on bets with a coefficient of 2 we have a 5% edge.

Decimal odds	Edge %
1.1	1.58
1.2	2.24
1.3	2.74
1.4	3.16
1.5	3.54
1.6	3.87
1.7	4.18
1.8	4.47
1.9	4.74
2	5
2.5	6.12

3	7.07
3.5	7.91
4	8.66
5	10
10	15
20	21.79

That is, according to this model, it follows that if a player is able to achieve an advantage of 5% on bets with a coefficient of 2, then at a coefficient of 1.2 he will most likely have an advantage of 2.24%.

Substituting the edge of $R\sqrt{(k-1)}$by the coefficient k in the formula of the optimal (according to Kelly) bet amount, we get the desired dependence of the bet amount on the coefficient. It will be like this: $S/\sqrt{(k-1)}$, where S is the sum of the bet on the coefficient 2. If we consider it equal to 5% of player bank, then the values of the bet amount for the remaining coefficients can be obtained from the table:

Decimal odds	Bet sum in % of bank
1.1	15.81
1.2	11.18
1.3	9.13
1.4	7.91
1.5	7.07
1.6	6.45
1.7	5.98
1.8	5.59
1.9	5.27
2	5
2.5	4.08
3	3.54
3.5	3.16
4	2.89
5	2.50
10	1.67
20	1.15

In accordance with the model under consideration, the amount of the bet on the 1.1 coefficient will not increase by 10 times (as in the fixed profit model), but only a little more than three times.

The third possible variant of the dependence of the player edge on the coefficient can be, purely formal, for example, such as: $R(k-1)$. This is a more 'strong' dependence. But for optimal Kelly sum it leads to constant, regardless of the coefficients, bets, which is obviously wrong.

Thus, the other two extreme options, in contrast to the option given in the tables, lead to 'impractical' results. This allows us to consider the considered model as a good approximation to reality. Moreover, it is built on the basis of some meaningful assumption.