The margin of the bookmaker.

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By margin, they usually mean a value that determines how much of the amount of bets placed by the players is taken by the bookmaker on average. If there are unlikely to be discrepancies in the definition of the substance, then there are disagreements when determining the formulas for margin for some reason.

On the Internet, on sports betting sites, there is such a formula for the margin of a bookmaker for a 3-way event:

$$m = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} - 1 \qquad (1)$$

Let's figure out what it gives and whether it gives the margin value correctly. First, consider betting on one outcome. Let the true value of the probability of the outcome of the event be *P*, and the coefficient by which we bet on the event is *K*. If we bet the amount *V* on the event, then on average we will lose (or receive, if there is a negative value) the amount:

$$S = (1 - P)V - P(K - 1)V \qquad (2)$$

That is, with probability (*1-P*) we lose the bet amount (the first part of the expression), and with probability *P* we have a profit of *(K-1)V* (the second part of the expression).

As a result, the mathematical expectation of our loss will be equal to *S*. If we attribute it to the bet amount *V*, then this will be the margin – that is, the part that we give (lose) bookmaker's office for every dollar bet. That is, *m = S/V* or *mV = S*. Substituting into the formula for *S* and simplifying it, we get the formula for margin through *K* and *P* for one outcome separately.

$$m = 1 - P * K \qquad (3)$$

Now let's assume that all three outcomes of the event have the same margin. That is, no matter what outcome we bet on, we will always lose the *m*-th part of the bet amount on average. If we if we bet on all three outcomes some amount in any proportion, then we will still lose on average the *m*-th part of the total amount bet. And, therefore, the margin for all three coefficients at once will also be equal to m. That is, if the margin for each outcome is equal to m, then the margin for all three outcomes at once is also equal to m, just in essence of what is happening. Now let's check this fact using the formula for margin (1), which is used everywhere.

To do this, we will substitute coefficients expressed in terms of probabilities and margin into it. We use formula (3) and express *K* in terms of *P* and *m*

$$K = (1 - m)/P$$

Substitute this expression with the same *m*, but different *K* (that is, different *P*) and we get

$$m = \frac{P_1}{(1-m)} + \frac{P_2}{(1-m)} + \frac{P_3}{(1-m)} - 1$$

Since the probabilities of the event here are 'true', their sum is equal to one. Therefore, the formula is simplified:

$$m = \frac{1}{(1-m)} - 1 = \frac{m}{(1-m)}$$

That is, we get a clear contradiction – the total margin is not equal to the margin for each of the outcomes. In order for there to be no contradiction, you need to change the margin formula (1) to this - you need to divide it by $\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$

Then we get a consistent formula for the 3-way margin:

$$m = 1 - \frac{1}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}} \quad (4)$$

If, as before, we substitute margin formulas for each of the outcomes (which are equal to each other and equal to *m*), then the total margin for the event will also be equal to m, as it should be in reality.

That is, the correct margin formula for a 3-way event will be formula (4). For 2-way and n-way events, everything is done similarly. The correct formula gives a slightly smaller margin value, since it differs from it by a divisor by a number greater than one. There will be a difference, but small, insignificant. But still it is there.

By the way, arbitrage services that show the percentage of scalps, and in fact, the negative margin of the combined (from several bookmakers where bets are placed on different outcomes) 'bookmaker', use formula (4), that is, the correct formula. And this is natural, since arbers count their real money and there should be no mistakes there.